

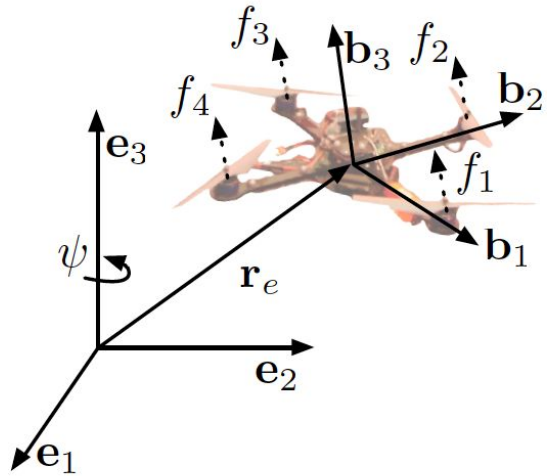
Quadrotor State Estimation and Obstacle Detection

Robot Autonomy Project
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- I. Dynamics**
- II. Differential Flatness**
- III. Planning**
- IV. Control Architecture**
- V. State Estimation (EKF)**
- VI. Sensors**
- VII. SLAM (RTAB Map)**
- VIII. Obstacle Detection**
- IX. Video**

Quadrotor Dynamics



Control inputs:

$$[f \ M]^T, f \in \mathbb{R}, M \in \mathbb{R}^3$$

State:

$$[x \ v \ R \ \Omega]^T$$

Dynamics:

$$\begin{aligned}\dot{x} &= v \\ m\dot{v} &= mge_3 - fRe_3 \\ \dot{R} &= R\hat{\Omega} \\ J\dot{\Omega} + \Omega \times J\Omega &= M\end{aligned}$$

Differential Flatness

Dynamics:

$$\begin{aligned}\dot{x} &= v \\ m\dot{v} &= mge_3 - fRe_3 \\ \dot{R} &= R\widehat{\Omega} \\ J\dot{\Omega} + \Omega \times J\Omega &= M\end{aligned}$$

Pick outputs: $y = y(x, u, \dot{u}, \dots, u^{(p)})$

Such that:

$$\begin{aligned}x &= x(y, \dot{y}, \dots, y^{(q)}) \\ u &= u(y, \dot{y}, \dots, y^{(q)}).\end{aligned}$$

Any 4 of the following 6 can serve as flat outputs:

X

Y

Z

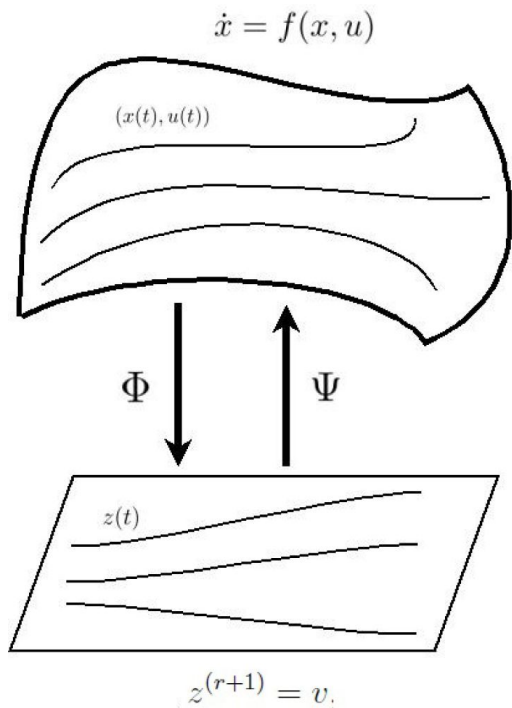
Phi

Theta

Psi

Murray, Richard M., Muruhan Rathinam, and Willem Sluis. "Differential flatness of mechanical control systems: A catalog of prototype systems." *ASME international mechanical engineering congress and exposition*. 1995.

Differential Flatness



Any 4 of the following 6 can serve as flat outputs:

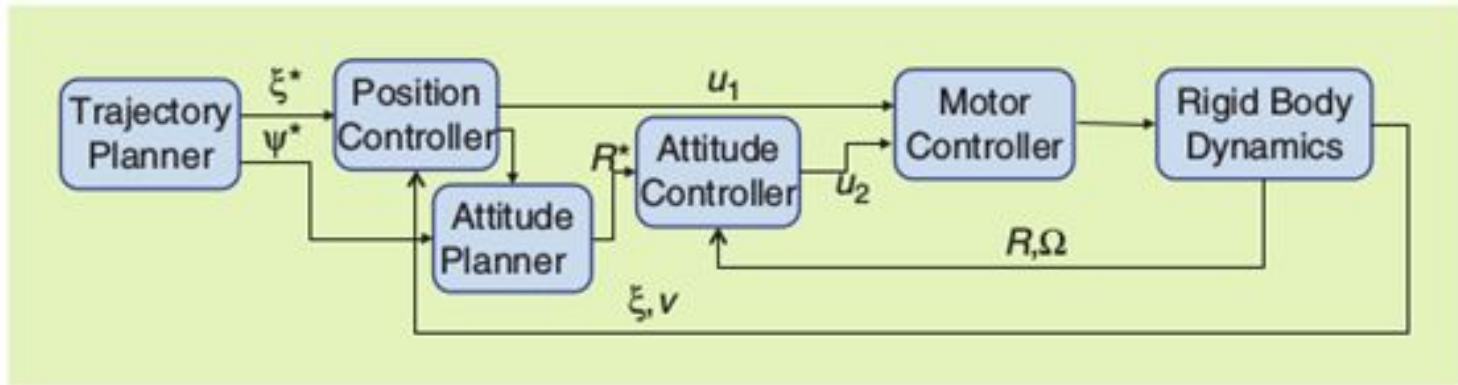
- X
- Y
- Z
- Phi
- Theta
- Psi

Trajectory Planning:

$$\min \int_{t_0}^{t_m} \mu_r \left\| \frac{d^{k_r} \mathbf{r}_{\mathbf{T}}}{dt^{k_r}} \right\|^2 + \mu_\psi \frac{d^{k_\psi} \psi_{\mathbf{T}}}{dt^{k_\psi}}^2 dt \quad k_r = 4, k_\psi = 2$$

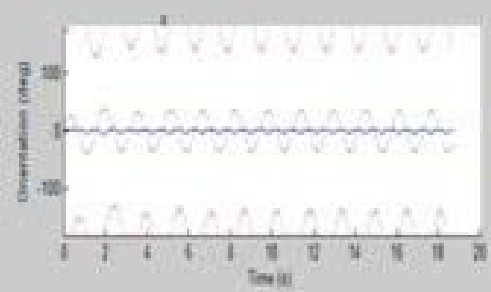
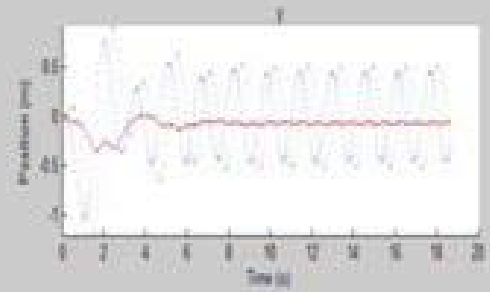
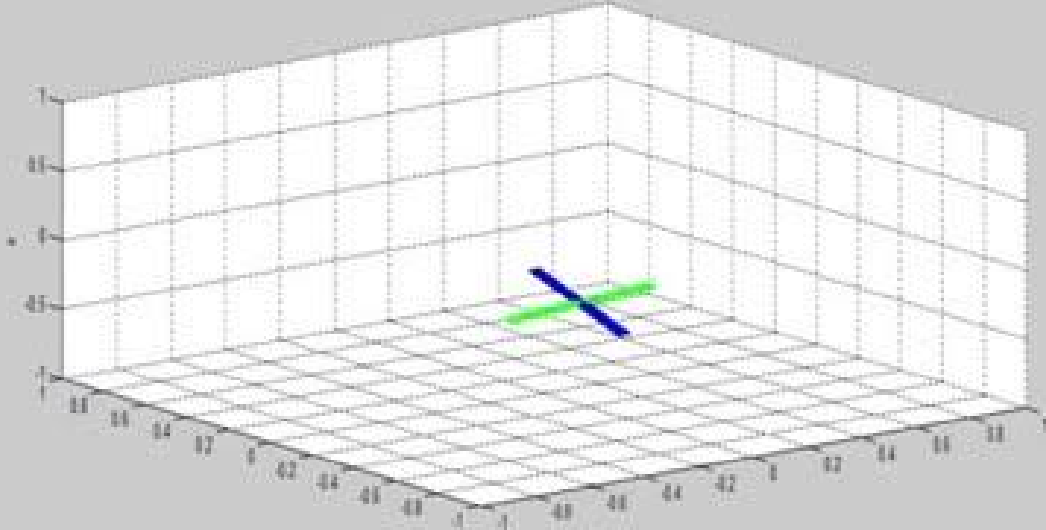
Mellinger, Daniel, Nathan Michael, and Vijay Kumar. "Trajectory generation and control for precise aggressive maneuvers with quadrotors." *The International Journal of Robotics Research* (2012): 0278364911434236.

CONTROL ARCHITECTURE



Mahony, Robert, Vijay Kumar, and Peter Corke. "Multirotor aerial vehicles: Modeling, estimation, and control of quadrotor." IEEE Robotics & Automation Magazine 19 (2012): 20-32.

Quadrotor Simulator

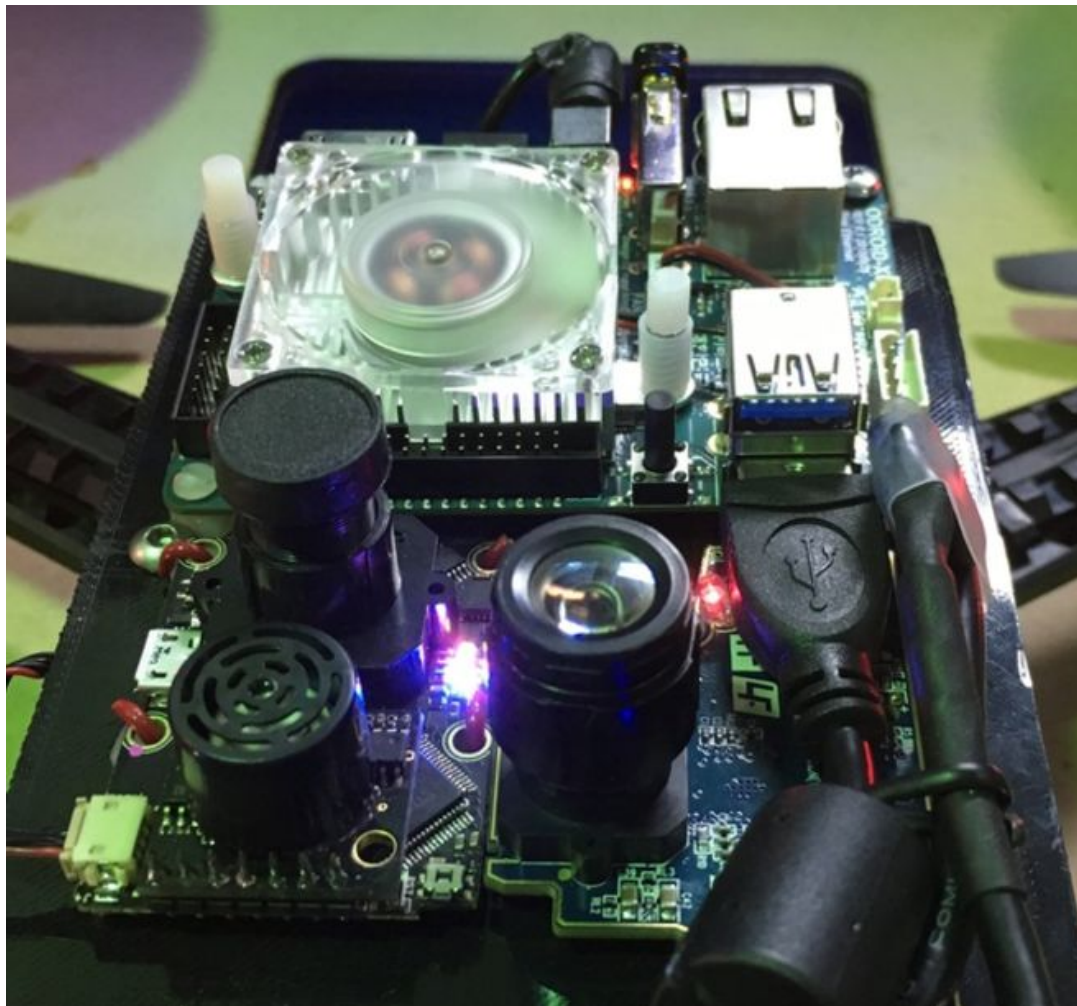
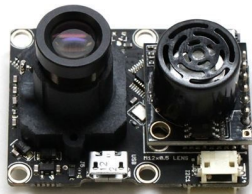


Sensors

Camera

IMU

Height
Sensor



Extended Kalman Filter

Kalman Filtering: Computations

Notation:

A_t : Motion Model

B_t : Control Input Model

μ_t : State Mean

Σ_t : State Variance

Q_t : Motion Model Noise

C_t : Observation Model

R : Observation Noise

K_t : Kalman Gain

$z_t - C_t\bar{\mu}_t$: Innovation

$$\text{KalmanFilter}(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$$

$$\bar{\mu}_t = A_t\mu_{t-1} + Bu_t$$

Prediction

$$\bar{\Sigma}_t = A_t\Sigma_{t-1}A_t^\top + Q_t$$

$$K_t = \bar{\Sigma}_t C_t^\top (C_t \bar{\Sigma}_t C_t^\top + R)^{-1}$$

Gain

$$\mu_t = \bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t)$$

Update

$$\Sigma_t = (I - K_t C_t)\bar{\Sigma}_t$$

Slide courtesy Kris Kitani

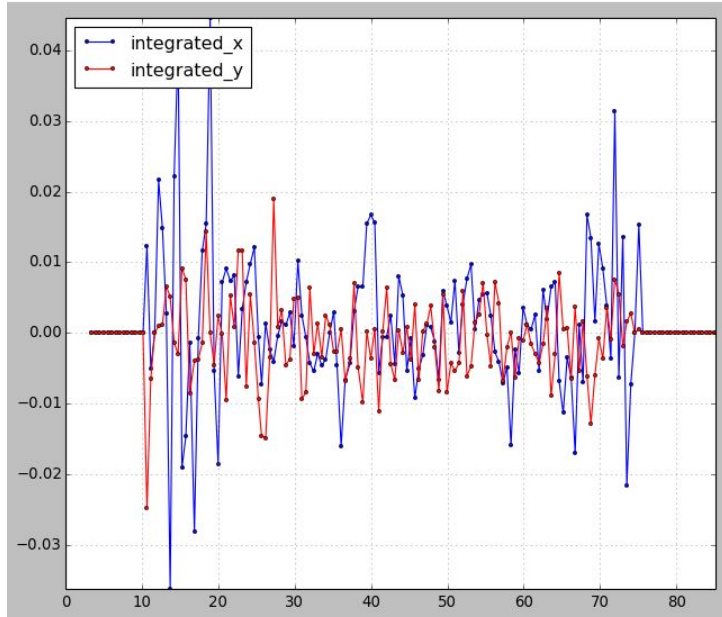
State Estimation with Optical Flow

$$v_x = \frac{T_z x - T_x f}{Z} - \omega_y f + \omega_z y + \frac{\omega_x x y - \omega_y x^2}{f}$$

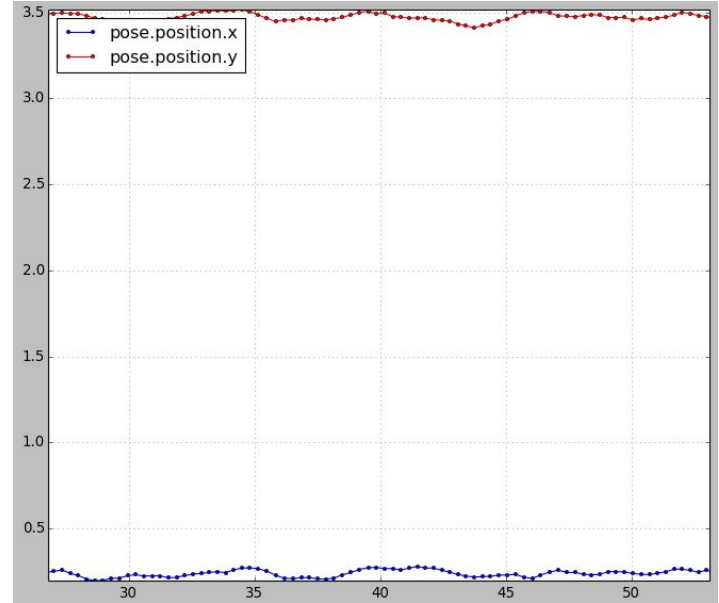
$$v_y = \frac{T_z y - T_y f}{Z} + \omega_x f - \omega_z x + \frac{\omega_x y^2 - \omega_y x y}{f}$$

$$\frac{\text{flow}}{\Delta \text{time}} = \mathbf{v} = f \frac{Z\mathbf{V} - V_z \mathbf{P}}{Z^2}$$

State Estimation with Optical Flow

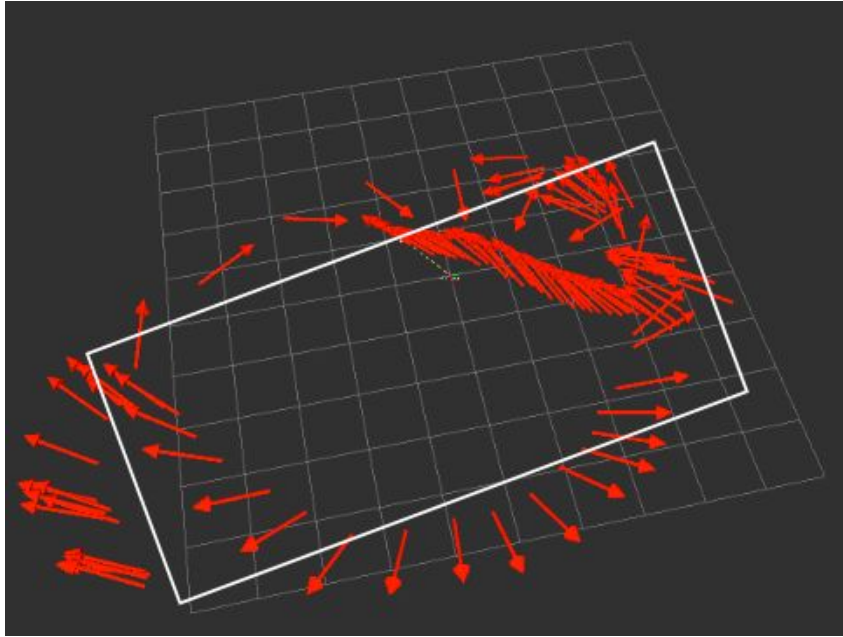


Velocity Updates from Optical Flow
Camera

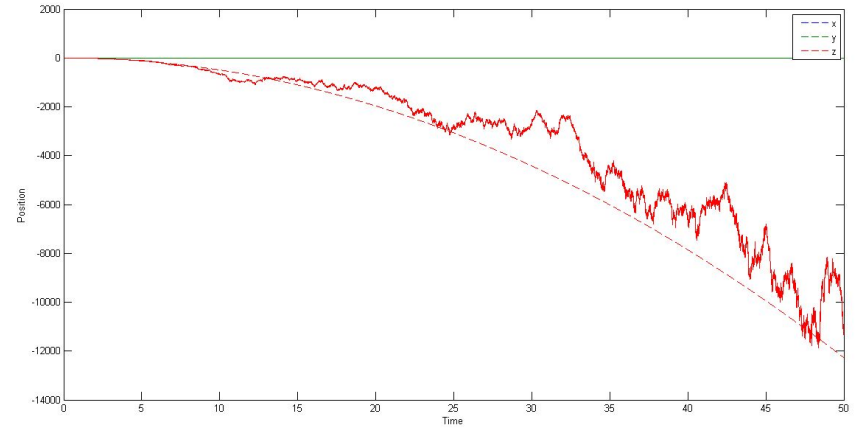


Position Updates from EKF

State Estimation with Optical Flow



Odometry Readings

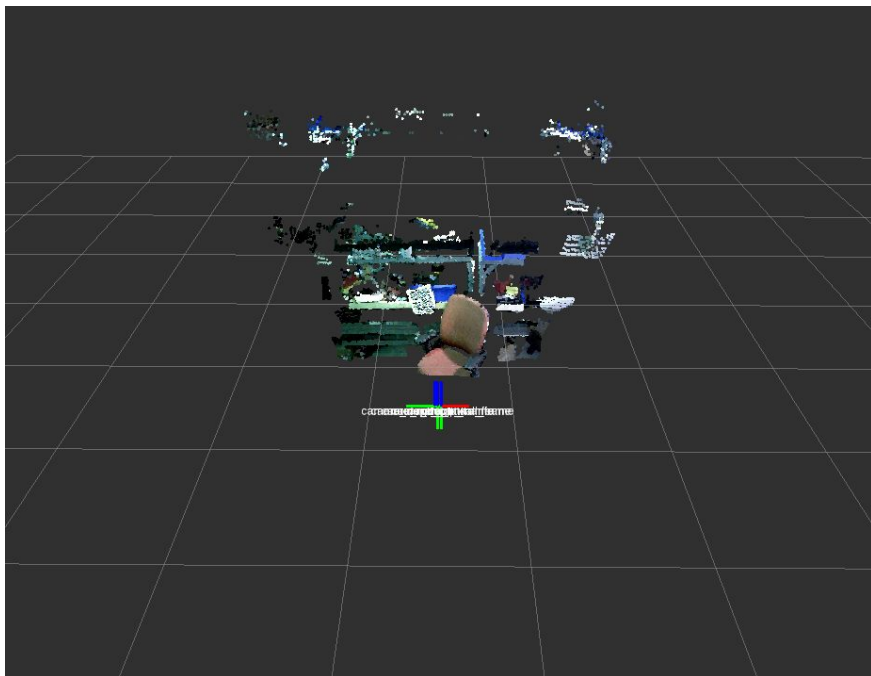


Linear Drift with Time in Simulation

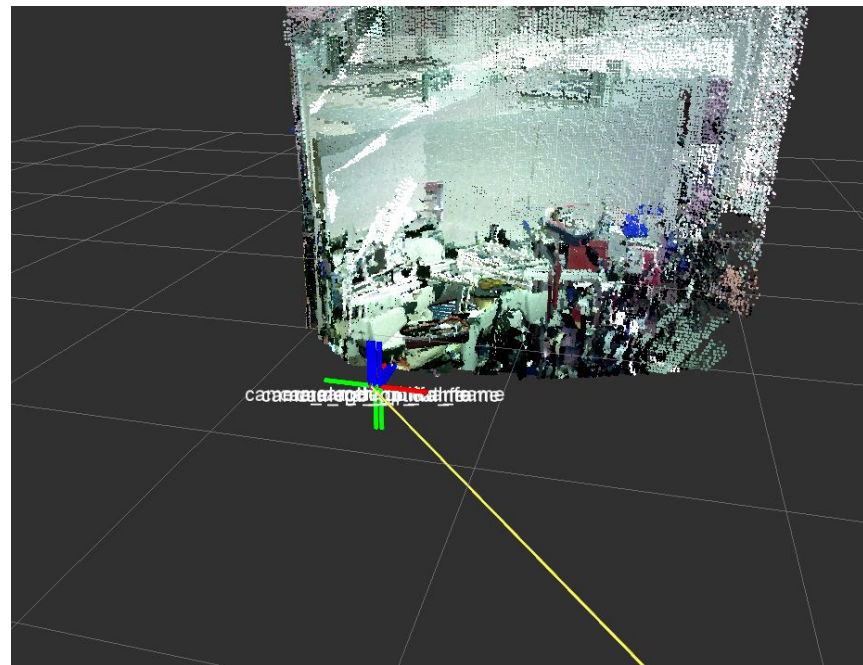
RTAB-Map

- Graph and Node based System
- Gathers RGB and Depth information
- OpenNI handles point clouds
- Uses visual words to detect loop closures

RGB-D SLAM

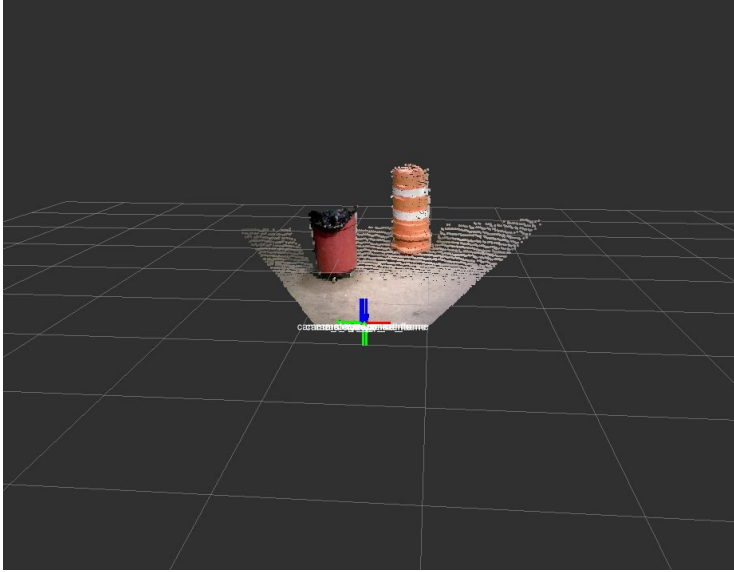


Static Map

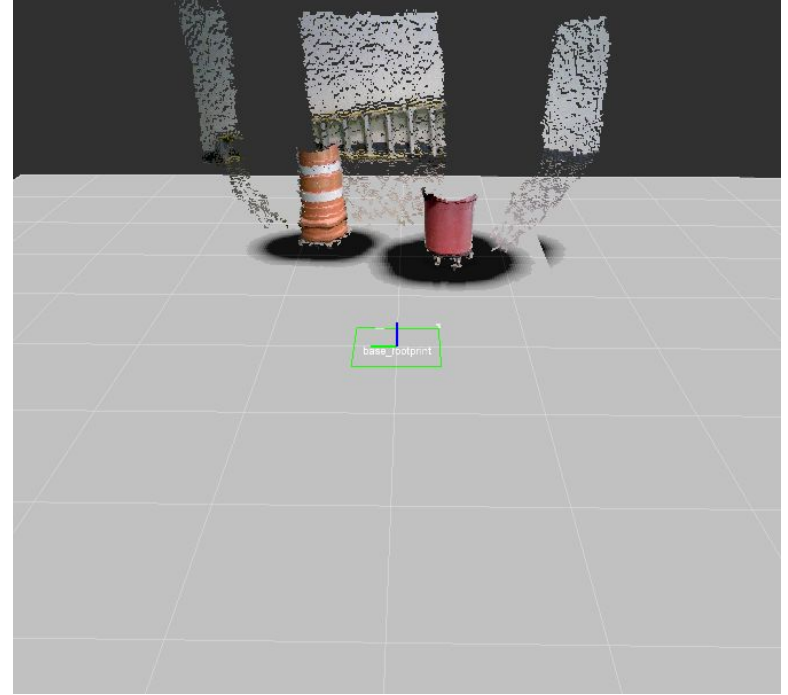


Dynamic Map Building

Obstacle Detection



RGB-D Point Cloud Data



Occupancy Grid

Visualization of Costmap and State Estimation

